

## LARRY'S LIST OF 10 BRAIN TEASERS

These brain teaser questions are indeed very interesting. Some of them are not easy to answer at all.

- (1) Given a  $3 \times 3$  square  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ , you are allowed to do circular shift on any row, and circular shift on any column, as many times as you please. Can you only switch position 1 and 2 with the allowed circular shift so as to get  $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ?
- (2) Visually there is a row of  $N$  bottles, which look exactly the same to each other. Among those  $N$  bottles, only 1 bottle is real and all the other bottles are just virtual images. You can detect the authenticity by touching the bottle. If you grab the fake one, the genuine one will switch the position either with its immediate left or right neighbor when available. Can you devise a mechanism to find the genuine bottle?
- (3) Suppose you are seeking long term relationship with your dream girl. You made your presence in the very popular TV show "Fei Cheng Wu Rao" ([http://en.wikipedia.org/wiki/Fei\\_Cheng\\_Wu\\_Rao](http://en.wikipedia.org/wiki/Fei_Cheng_Wu_Rao)) and posted your profile on the online dating site (<http://zhenai.com/>). Because you are so attractive, you have received quite a few correspondences. You have never ever got a chance to know any of these candidates so you plan on dating like  $n = 10$  girls. Because you are very serious and would show your sincere respect to each girl you want to meet, this process is supposed to be held sequentially, i.e., concurrent dating is not allowed (you are not a philander and would not take risks getting crazy about running for dating events at all times). God puts you in the following situation: 1) You are lucky enough that your universally unique dream girl is among the  $n$  people. 2) For any two girls you met already, you can tell which one suits you better. 2) In the dating process, as of today, some girl in your mind is the current best. But you are not sure if she is the best of all since your dream girl may still be lined up. 3) If you decide to move on to dating the next girl, you will never be able to date the girls you abandoned. (They are all very charming, decent and elegant. Self-esteem won't allow them to date you again once they got rejected by you.) Okay, now you are in a dilemma. There is no doubt you want to find your dream girl. However, if you go with some girl at a very early stage in this dating process, what a pity you lose the opportunity that might occur only once in your lifetime to find your "Ms. Right". In contrast, if you hesitate to choose, as time advances, you may find the quality of the girls is relatively deteriorating. You may regret what a fool you are as a result of undue greediness in hope of finding the best of best among all girls. The question is: what should be considered a smart move to find your dream girl from probability's perspective? In another words, when should be an optimal stopping time for the dating process such that you will choose the right girl with maximum probability?
- (4) A scientific calculator's digit keypad and  $+, -, *, \backslash$  are malfunctioning whereas  $\exp, \ln, X^2, \text{sqrt}, \sin, \cos, \tan, n!, 1/x$  and the inverse function key are just working out fine. This calculator is memoryless so how would you get 2 and 3 when the initial number on display is 0? What if you were asked to generate all natural numbers?

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- (5) You have a pile of 1000 stones. You split this pile into two smaller piles, not necessarily of the same size. You multiply the number of stones in the first pile with the number of stones in the second pile and write down the product on a sheet of paper. From the two piles of stones you choose one and split it into two smaller piles. Again you calculate the product of the numbers of stones in the two piles and write it down on the same sheet of paper. Now you have three piles. You choose one... You continue splitting in whatever manner suits you best until you are left with exactly 1000 piles with exactly 1 stone in each pile. On the sheet of paper, by that time, you have a list of numbers. You sum up those numbers. Prove that the result of the summation does not depend on the way of splitting. Calculate the result of the summation. (Continuous version: A string of length 1, cut it to two strings of length  $x$  and  $y$ , you get a number  $xy$ . Then you do similar splitting and add all the numbers up. What is the limit of the sum?)
- (6) 100 tigers and 1 sheep on a island only has grass. tiger can eat grass, but they would rather choose to eat sheep. Two conditions: A, each time only one tiger eat one sheep, than itself become a sheep. B, the tigers are smart and can think logically. Question is, will the sheep be eaten finally?
- (7) A remote island has three types of chameleons with the following population: 13 red chameleons, 15 green chameleons and 17 blue chameleons. Each time two chameleons with different colors meet, they would change their color to the third color. For example, if a green chameleon meets a red chameleon, they both change their color to blue. Is this ever possible for all chameleons to become the same color? Why or why not?
- (8) Toss a fair coin repeatedly and stop whenever you want, receiving as a reward the average number of heads accrued at the time you stop. If your first toss is a head, and you stop, your reward is 1 Krugerrand. Since you can never have more than 100 percent heads, it is clearly optimal to stop in that case. If the first toss is a tail, on the other hand, it is clearly best not to stop, since your reward would be zero..., when would you stop then?
- (9) Marvin gets off work at random times between 3 and 5 P.M. His mother lives uptown, his girlfriend downtown. He takes the first subway that comes in either direction and eats dinner with the one he is first delivered to. His mother complains that he never comes to see her, but he says she has a 50 – 50 chance. He has had dinner with her twice in the last 20 working days. Explain why this could happen? ( $\frac{1}{10}$  instead of  $\frac{1}{2}$ )
- (10) (I love this puzzle: This puzzle explores the issues of altruism, loyalty, and self-interest.) A young man is driving along in his two-seater sports car on a stormy night. He passes a bus stop and sees three people waiting for the bus: 1) An elderly woman who is in serious distress and needs to get to the hospital or she will die. 2) A best friend from the Army who once saved his life. 3) An attractive woman who may be the young man's soul-mate. The young man's car accommodates only two people. What do you think he should do in this situation?